INVESTIGATION OF THE INFLUENCE OF GAS MICRONUCLEUS FORMATION ON PERTURBATION PROPAGATION IN GAS-LIQUID SYSTEMS

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Results are presented of an investigation of perturbation propagation in gas-saturated systems with gas micronuclei formation taken into account.

Investigation of the motion of gas-liquid systems in domains above and close to saturation pressure reduces, as a rule, to modelling homogeneous media. This is related to the fact that the transition from one state to another occurs instantaneously in the equilibrium thermodynamic theory of phase transformations.

However, according to [1], the formation of a new phase does not occur instantaneously and the nuclei newly being formed (gas bubbles) are "heterophasal" while the system under consideration is assumed not completely homogeneous. A "heterophasal" system can be in both the equilibrium and nonequilibrium states.

It was established by experimental investigations [2-4] that the level and tempo of the pressure change affect micronuclei formation. It is also established that at pressures above the saturation pressure in gas-liquid systems hysteresis phenomena are observed on the flow curves which, when taken into account, can result in qualitatively new effects for different transient motion modes.

This paper is devoted to an investigation of perturbation propagation in gas-saturated systems with gas micronuclei formation taken into account. The mathematical model proposed in [4] for the motion of a gas-liquid medium in pipes is used here, which reflects the process being studied sufficiently well. Certain extensions of the model mentioned are also proposed.

1. The system of differential equations describing the motion of a gas-liquid system is written analogously to [5]

$$\frac{\partial (f\rho w)}{\partial t} + \frac{\partial (f\rho w^2)}{\partial x} = -f\frac{\partial P}{\partial x} + \chi\tau, \quad \frac{\partial (f\rho)}{\partial t} + \frac{\partial (f\rho w)}{\partial x} = 0. \tag{1}$$

After some manipulations, it can be represented as follows

$$\frac{\partial w}{\partial t} + \frac{1}{2}\frac{\partial w^2}{\partial x} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} + \frac{\chi}{\rho_0 f_0}\tau(w), \quad \frac{\partial (f\rho)}{\partial t} = -\frac{\partial (f\rho w)}{\partial x}. \tag{2}$$

Giving linear laws of the change in density and pipe area due to pressure, we can write the second equation in (2) as

$$\frac{\partial p}{\partial t} = -\rho_0 c_0^2 \frac{\partial w}{\partial x}, \tag{3}$$

where

$$c_0^2 = k/\rho_0; \quad k = k_m \Big/ \left(1 + a_0 \frac{k_m}{E}\right); \quad a_0 = d/\delta_0.$$

The solution of the system of nonlinear equations for a given friction law and with gas inclusions taken into account can be realized by numerical methods. The approximate solution of the system (the first equation in (2) and (3)) can be obtained analogously to [6].

Using a number of transformations and approximations with (3) taken into account we write the first equation ($\partial/\partial t \approx -c_0\,\partial/\partial x$, $P \approx \rho_0 c_0 w$) in (2) (the motion equation) in the form

$$\frac{\partial w}{\partial t} + (c_0 + w)\frac{\partial w}{\partial x} = \beta_0 \tau(w), \tag{4}$$

where $\beta_0 = I/(\rho_0\delta)$; $\delta = f_0/\chi$.

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The equation (4) for the pressure function P can be represented as

$$\frac{\partial P}{\partial t} + (c_0 + \alpha_0 P)\frac{\partial P}{\partial x} = \beta_1 \tau(P), \tag{5}$$

where $\alpha_0 = 1/(\rho_0 c_0)$; $\beta_1 = c_0/\delta$.

Since the flow curve of gas-liquid systems can be represented as a function of the gas inclusion concentration C and the velocity w, i.e., ω, i.e. $\tau(C, \omega)$, then (4) and (5) are written as

$$\frac{\partial}{\partial t}\begin{Bmatrix} P \\ w \end{Bmatrix} + \left(c_0 + \begin{Bmatrix} \alpha_0 P \\ w \end{Bmatrix}\right)\frac{\partial}{\partial x}\begin{Bmatrix} P \\ w \end{Bmatrix} = \begin{Bmatrix} \beta_1 \\ \beta_0 \end{Bmatrix}\tau\left(C, \begin{Bmatrix} P \\ w \end{Bmatrix}\right). \tag{6}$$

For (6) to be solvable, it is necessary to have an equation in the stress τ and the concentration C that can be represented in the generalized form

$$\frac{d\tau}{dt} = f_1(\tau, C, w, \ldots), \tag{7}$$

$$\frac{dC}{dt} = f_2(\tau, C, w, \ldots). \tag{8}$$

The relationships (6)-(8) obtained form a complete system of motion equations for gas-liquid systems in pipes in the presence of gas micronuclei.

2. Let the dependence of the stress on the concentration and velocity be given analogously to [4]

$$\tau = -\frac{4\mu_0}{R}(1 + \gamma C)w. \tag{9}$$

Assuming the value of the concentration to depend on the pressure level, the tempo of the pressure change, and the nonequilibrium of extraction and dissolution of the gas phase, (8) can be written in the form [4]

$$C = \beta_2(P - P_b) - \frac{\beta_2(\theta - \lambda)}{\theta}\int_0^t \exp\left(-\frac{t - t_1}{\theta}\right)\frac{dP}{dt_1}dt_1. \tag{10}$$

Since the processes are examined at the pressure $P = P_b$, then to $\beta_2 < 0$.

Taking (10) and the approximation presented above into account, (9) can be represented in the form

$$\tau = -b_0 P - b_1 P^2 + b_2 P\int_0^t \exp\left(-\frac{t - t_1}{\theta}\right)\frac{dP}{dt_1}dt_1, \tag{11}$$

where

$$b_0 = \frac{4\mu_0}{R\rho_0 c_0}(1 - \gamma\beta_2 P_b); \quad b_1 = \frac{4\mu_0\gamma\beta_2}{R\rho_0 c_0}; \quad b_2 = \frac{4\mu_0\gamma}{R\rho_0 c_0}\frac{\beta_2(\theta - \lambda)}{\theta}.$$

We realize the solution of the system (5) and (11) after certain simplification. If it is assumed that the function dP/dt_1 varies mainly because of the convective component, then it can be represented in the form $dP/dt_1 \approx -c_0 \partial P/\partial x$. The system (5) and (11) in P is then written as

$$\frac{\partial P}{\partial t} + (c_0 + \alpha_0 P)\frac{\partial P}{\partial x} = -\alpha_1 P - \alpha_2 P^2 - \alpha_3 P\frac{\partial P}{\partial x}, \tag{12}$$

where $\alpha_1 = b_0\beta_1$; $\alpha_2 = b_1\beta_1$; $\alpha_3 = b_2\beta_1\theta c_0$.

The solution of (12) along the characteristics [7] under the condition that at the initial time $t = 0$, $x = \xi$ a certain perturbation $P = P_0(\xi)$ is given has the following form

$$\tilde{P} = \frac{P}{P_0(\xi)} = \frac{\exp(-\alpha_1 t)}{1 + \frac{\alpha_2}{\alpha_1}P_0(\xi)(1 - \exp(-\alpha_1 t))}, \tag{13}$$

$$x = \xi + c_0 t + \frac{\alpha_0 + \alpha_3}{\alpha_2}\ln\left[1 + \frac{\alpha_2}{\alpha_1}P_0(\xi)(1 - \exp(-\alpha_1 t))\right]. \tag{14}$$

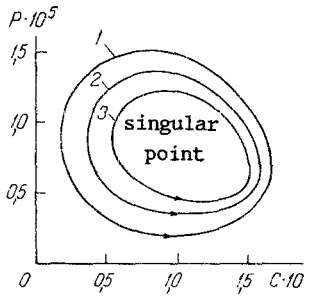

Fig. 1

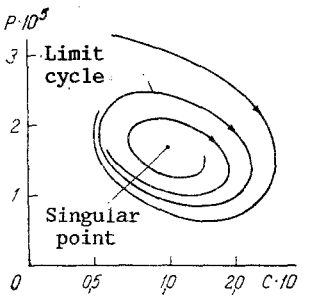

Fig. 2

Fig. 1. Closed trajectories corresponding to solutions of the Lotky—Volterra system ($P_b = 1.5 \cdot 10^5$ Pa; $\beta^0 = 0.0001$ m^2/N; $\theta = 10$ sec; $\gamma_0 = 10$; $\mu_0 = 0.01$ Pa·sec; $\mu_c = 0.0095$ Pa·sec; $\rho_0 = 1000$ kg/m^3; $c_0 = 1370$ m/sec): I) $P(0) = 1.3 \cdot 10^5$ Pa; $C(0) = 0.12$; 2) $P(0) = 0.6 \cdot 10^5$ Pa; $C(0) = 0.1$; 3) $P(0) = 1.0 \cdot 10^5$ Pa; $C(0) = 0.1$ ($P' \cdot 10^{-5}$ should be read on the ordinate axis in Figs. 1 and 2).

Fig. 2. Solution of limit cycle type: $V_0 = 0.002$ 1/sec; $\kappa_0 = 0.1$ 1/sec; $\kappa_1 = 0.05$ 1/sec; $\kappa_2 = 0.02$ 1/(Pa·sec); $\gamma_{01} = 10.1$; $\gamma_{11} = 25$; $P(0) = 2 \cdot 10^5$ Pa; $C(0) = 0.15$.

Analysis of the relationship (13) shows that the pressure change depends on the initial perturbation $P_0(\xi)$, the pressure of micronuclei formation P_b, the constants γ, β_2 that take account of the presence of micronuclei, where as $P_0(\xi)$ increases the perturbation damping in the system occurs more rapidly while as the parameters P_b, γ, β_2 increase (diminution of α_1) the value of the pressure damps out considerably more slowly than for fluids without a gas micronuclei inclusion content.

It is seen from (14) that the wave velocity depends substantially on the time θ of micronuclei formation and the tempo λ of the pressure change. The presence of the parameter θ increases the wave propagation velocity and the influence of the nonlinearity on the perturbation wave propagation velocity while the parameter λ diminishes the wave velocity and the influence of the nonlinearity.

The possibility of wave flip-flop can be estimated on the basis of differentiating (14) with respect to the parameter ξ or analogously to [6].

3. Nonstationary motion of an incompressible gas-liquid medium is considered. In this case the process can be described by the first equation of the system (2) without taking account of the convective component. Assuming the stress dependence on the concentration to correspond to (9) and the pressure gradient proportional to the velocity ($\partial P/\partial x = -P'/l = -(8\mu_c/R^2)\, w$), the motion equation (2) is representable after simple manipulations as

$$\frac{d}{dt}\begin{Bmatrix} P' \\ w \end{Bmatrix} = -\frac{8\mu_0}{\rho_0 R^2}\left(1 - \frac{\mu_c}{\mu_0}\right)\begin{Bmatrix} P' \\ w \end{Bmatrix} - \frac{8\mu_0\gamma}{\rho_0 R^2} C \begin{Bmatrix} P' \\ w \end{Bmatrix}. \tag{15}$$

It is also assumed that the velocity of micronuclei formation is proportional to the difference between the pressure P and the pressure of the beginning of nuclei formation P_b, where the proportionality factor depends on the concentration C in such a manner that the value of the factor diminishes as C increases.

Taking the above elucidation into account, the equation describing the change in micronuclei concentration can be represented as follows:

$$\theta \frac{dC}{dt} + C = \beta(C)(P - P_b). \tag{16}$$

Since it was assumed that the coefficient of nuclei-formation diminishes as C increases, the dependence $\beta(C)$ can be written in the form $-\beta^0 C$, where the parameter γ is assumed to equal $-\gamma_0$.

In connection with the above, the system (15) and (16) for the parameters P′ and C takes a form analogous to a Lotky—Volterra system

$$\frac{dC}{dt} = k_1 C - k_2 P' C, \tag{17}$$

$$\frac{dP'}{dt} = k_3 C P' - k_4 P', \tag{18}$$

where

$$k_1 = \frac{(\beta^0 P_b - 1)}{\theta};\ k_2 = \frac{\beta^0}{\theta};\ k_3 = \frac{8\mu_0\gamma_0}{\rho_0 R^2};\ k_4 = \frac{8\mu_0}{\rho_0 R^2}\left(1 - \frac{\mu_c}{\mu_0}\right).$$

The system (17) and (18) has a single non-zero stationary solution

$$P^0 = \frac{k_1}{k_2} = \frac{\beta_0 P_b - 1}{\beta^0}, \tag{19}$$

$$C^0 = \frac{k_4}{k_3} = \frac{1}{\gamma_0}\left(1 - \frac{\mu_c}{\mu_0}\right). \tag{20}$$

The method of normal modes [8] is utilized to investigate the stability of the stationary state of the differential equations (17) and (18). Consequently, the dispersion equation takes the form

$$\omega^2 + k_1 k_4 = 0. \tag{21}$$

The stability is associated with the sign of the real parts of the roots of the dispersion equations. In the case when $\beta^0 P_b - 1 < 0$, $\mu_c < \mu_0$ or $\beta^0 P_b - 1 > 0$, $\mu_c > \mu_0$, the unperturbed state is stable for the solution $\omega_n < 0$. When $\beta^0 P_b - 1 > 0$, $\mu_c < \mu_0$ or $\beta^0 P_b - 1 < 0$, $\mu_c > \mu_0$, the real parts vanish, $\mathrm{Re}\,\omega_n = 0$, for each solution while the imaginary parts equal

$$\mathrm{Im}\,\omega_n = \pm (k_1 k_4)^{1/2}. \tag{22}$$

In this case neutral stability holds around the stationary state (19) and (20), where the frequency of rotation (oscillation) corresponds to the limit of small perturbations and depends on the amplitude. Characteristic phase trajectories of periodic solutions are represented in Fig. 1 for different initial values.

4. In conformity with the main representation about the heterophasal fluctuations it is assumed that the formation of gas micronuclei for metastable processes depends in a substantial measure on the rates of origination, growth, and disappearance of nuclei of the newly occurring forms [1]. It can here be assumed that the rate of gas inclusion formation is proportional to the square of the concentration (this corresponds to a probability of pairwise cooperation of micronuclei, where the proportionality factor depends on the pressure) while the rate of bubble disappearance (dissolution) is proportional to their concentration.

The differential equation describing the change in concentration under the assumptions made has the form

$$\frac{dC}{dt} = V_0 - \varkappa_0 C + \varkappa(P')\,C^2. \tag{23}$$

The quantity $\kappa(P')$ characterizes the micronuclei cooperation factor. Since it is assumed that the probability of micronuclei connection diminishes as the pressure increases, then $\kappa(P')$ can be represented as $\kappa(P') = \kappa_1 - \kappa_2 P'$. Then (23) is written as

$$\frac{dC}{dt} = V_0 - \varkappa_0 C + \varkappa_1 C^2 - \varkappa_2 C^2 P'. \tag{24}$$

The dependence of the shear stress on the concentration and velocity can here be represented as follows

$$\tau = -\frac{4\mu_0}{R}(1 - \gamma_0 C + \gamma_1 C^2)\,w. \tag{25}$$

Then taking account of (25) and the assumptions made above, the equation (2) of fluid motion takes the form

$$\frac{dP'}{dt} = -k_4(1 - \gamma_{01} C + \gamma_{11} C^2)\,P', \tag{26}$$

where $\gamma_{01} = k_3/k_4$, $\gamma_{11} = 8\mu_0\gamma_1/(\rho_0 R^2 k_4)$.

The system (24) and (26) allows of four stationary states

$$P^s_{1,2} = 0,\quad C^s_{1,2} = \frac{\varkappa_0 \pm \sqrt{\varkappa_0^2 - 4\varkappa_1 V_0}}{2\varkappa_1}, \tag{27}$$

$$C^s_{3,4} = \frac{\gamma_{01} \pm \sqrt{\gamma_{01}^2 - 4\gamma_{11}}}{2\gamma_{11}},\quad P^s_{3,4} = \frac{V_0 - \varkappa_0 C^s_{3,4} + \varkappa_1 (C^s_{3,4})^2}{\varkappa_2 (C^s_{3,4})^2}. \tag{28}$$

From physical considerations it is assumed that the quantities C^s and P^s have real and positive values. Applying the method of normal modes, as above, it is easy to obtain the dispersion equation

$$\omega^2 + (\varkappa_0 - 2\varkappa_1 C^s + 2\varkappa_2 P^s C^s)\,\omega + k_4 \varkappa_2 (\gamma_{01} - 2\gamma_{11} C^s) P^s = 0. \tag{29}$$

Taking account of the stationary states, an analysis of (29) permits considering that the possibility of limit cycle origination in a "brusselator" can be realized for the following stationary state

$$C^s = \frac{\gamma_{01} - \sqrt{\gamma_{01}^2 - 4\gamma_{11}}}{2\gamma_{11}}, \quad P^s = \frac{V_0 - \varkappa_0 C^s + \varkappa_1 (C^s)^2}{\varkappa_2 (C^s)^2}.$$

In this case the dispersion equation (29) takes the form

$$\omega^2 + \frac{2V_0 - \varkappa_0 C^s}{C^s}\omega + \frac{k_4}{(C^s)^2}(\gamma_{01}^2 - 4\gamma_{11})^{1/2}[V_0 - \varkappa_0 C^s + \varkappa_1 (C^s)^2] = 0. \tag{30}$$

It is easy to see that the real part of one of the roots becomes positive if $\kappa_0 C^s > 2V_0$, where this latter corresponds to the instability condition in the brusselator circuit [8, 9].

The qualitative analysis performed on the system (24) and (26) as well as the dispersion equation (30) permits the assertion that the system has a limit cycle under definite conditions, i.e., any initial point in the P'C plane approaches the identical periodic trajectory with time. The characteristic curves are represented in Fig. 2.

Trajectories that correspond to the periodic solution that does not belong to a continuous family of closed trajectories and, as a rule, possess a structurally stable neighborhood are a stable limit cycle of an autonomous system.

NOTATION

f, area of the pipe transverse section; ρ, fluid density; w, mean fluid velocity over the section; P, pressure; P', pressure drop; P_b, pressure at which the first gas micronuclei are formed in the fluid; χ, wetted perimeter; τ, tangential stress; x, motion coordinate; t, time; k_m, system bulk compression modulus; E, elastic modulus of the pipe material; δ_0, pipe wall thickness; ρ_0, liquid density at the pressure P_0; c_0, wave velocity at the pressure P_0; μ_0, fluid viscosity; μ_c, system viscosity; R, pipe radius; γ, constant; θ, time characterizing the micronuclei formation; λ, time characterizing the pressure change tempo; β_2, a constant; C, gas inclusion concentration; V_0, micronuclei formation rate; κ, micronuclei disappearance factor. Indices: s, stationary state; m is mixture, and b is beginning of nuclei formation.

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